A study on using modal parameters and vibration test data to update dynamic model

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Abstract: This paper presents a synthesized model updating method using the modal parameters and frequency response functions(FRF) extracted from the vibration test data. Based on an established preliminary dynamic model, this method first updates the analytical model by use of the measured modal parameters, and then updates the dynamic characteristics of the analytical model with the measured frequency response functions in order that the modal parameters and FRF acceleration obtained by the updated dynamic model agree well with those measured. The detailed mathematical theory and formulation of this method are presented in the paper. The validation and accuracy of this method is analyzed and discussed through a numerical example.

Key words: dynamic model; vibration test; modal parameters; frequency response functions
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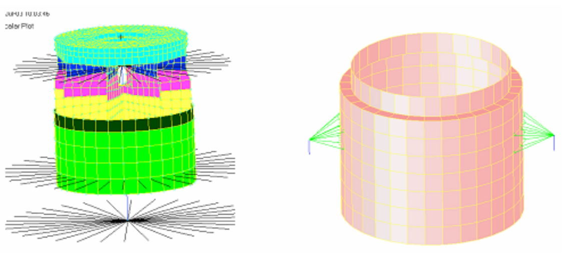
1 Introduction

The simulation of a vibration testing is very useful in the environment test for optimizing the design of the vibration test fixture, for predicting the vibration test results for a complex structure, for improving the test quality, and also for estimating the response for locations that are not measured during the vibration test, etc.

One of the key techniques in the vibration testing simulation is to set up an accurate dynamic model of the shaker test system. There are two steps in seting up an accurate dynamic model: the first step is to build the right FEA model of the shaker test system (see Figure 1); the second step is to update the dynamic model by using the test data in order to make the model have the dynamic characteristics of the shaker test system. The right simulation analysis results of vibration testing can be obtained only when the dynamic model has the real physical properties of the vibration testing system, so a good model improvement method is of great importance.



(a) The armature parts　　(b) The still parts

Fig.1　FEA model of the shaker system

Various model-updating methods have been developed in recent years for correcting analytical models to predict test results more closely[1-9]. Generally, the model updating methods can be divided into two groups: one is called the modal method, the other is called the Frequency Response Functions (FRF) method. The modal method can make modal parameters such as nature frequencies and mode shapes obtained from the updated model conform to those measured. The FRF method can improve the FRFs of the analytical model with those measured. Though the two methods update a dynamic model in different ways, they all have disadvantages. The modal method can update modal parameters of an analytical model efficiently, but not FRFs, which can make FRFs deviate from those measured. The FRF method, on the other hand, can improve the response of the analytical model, but does not update modal parameters.

In order to make full use of vibration test data, it is important to update both the modal parameters and FRFs of an analytical model, and to make the modal parameters and FRFs of the updated model consistent with those measured. Based on this idea, this paper proposes a synthesized model updating method using both the modal parameters and frequency response functions extracted from the vibration test data.

2 Modal Updating Method

An undamped n-degree of freedom system is governed by the equation of motion[10]

$$M_A\{\ddot{x}\}+K_A\{x\}=\{0\} \quad , \tag{1}$$

in which M_A, K_A are the system's mass and stiffness matrices, respectively.

It is assumed that the natural frequencies Λ_A and mode shapes Φ_A of the analytical model of n degrees of freedom are incorrect and that we have the corresponding test frequencies Λ_t and mode shapes Φ_t. The subscripts t and A represent the test model and the analytical model,

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respectively. The difference of modal parameters between the test model and the analytical model is as follows

$$\left.\begin{aligned}\Delta\Lambda &= \Lambda_t - \Lambda_A \\ \Delta\Phi &= \Phi_t - \Phi_A\end{aligned}\right\}. \quad (2)$$

Assuming that $\Delta\Lambda$ and $\Delta\Phi$ are small, namely that they can be considered as a perturbation, and introducing a small parameter ε, we have:

$$\left.\begin{aligned}\Lambda_t &= \Lambda_A + \varepsilon\Delta\Lambda \\ \Phi_t &= \Phi_A + \varepsilon\Delta\Phi\end{aligned}\right\} \quad (3)$$

As can be seen from Eq. (3), when $\varepsilon = 0$, we have $\Lambda_t = \Lambda_A$ and $\Phi_t = \Phi_A$, which represents the analytical values; when $\varepsilon = 1$, it represents the test values. In order to update mass matrices M_A and stiffness matrices K_A of the analytical model, we assume that the right mass and stiffness matrices can be expressed in power series of ε as follows:

$$M_t = M_A + \varepsilon\Delta M_1 + \varepsilon^2\Delta M_2 + \cdots, \quad (4a)$$

$$K_t = K_A + \varepsilon\Delta K_1 + \varepsilon^2\Delta K_2 + \cdots. \quad (4b)$$

When $\varepsilon = 0$, the above equations represent the original mass and stiffness matrices before updating, and when $\varepsilon = 1$, we obtain the desired result. Due to the orthogonality of mode shapes,we have:

$$\Phi_t^T M_t \Phi_t = \mathrm{I} \quad , \quad (5)$$

$$K_t\Phi_t = M_t\Phi_t\Lambda_t \quad . \quad (6)$$

First, we improve the mass matrices of the analytical model. Substituting Eqs. (3) and (4a) into Eq. (5), we have

$$(\Phi_A + \varepsilon\Delta\Phi)^T(M_A + \varepsilon\Delta M_1 + \cdots)(\Phi_A + \varepsilon\Delta\Phi) = \mathrm{I}. \quad (7)$$

Comparing the terms with the same power of ε on both sides of the above equation, we obtain the matrix equation for the perturbation method as follows:

$$\Phi_A^T M_A \Phi_A = I, \quad (8)$$

$$\Phi_A^T M_A\Delta\Phi + \Phi_A^T\Delta M\Phi_A + \Delta\Phi^T M_A\Phi_A = 0. \quad (9)$$

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Obviously, Eq. (8), which is the eigenvalue equation of the analytical model, is satisfied. From Eq. (9) we have:

$$\Phi_A^T\Delta M\Phi_A = -(\Phi_A^T M_A\Delta\Phi + \Delta\Phi^T M_A\Phi_A), \quad (10)$$

in which $\Delta\Phi$, Φ_A and M_A are known quantities. Thus, the mass matrix correction of the analytical model ΔM can be calculated from this equation.

Next, we update the stiffness matrices of the analytical model. Substitution of Eqs. (3) and (4b) into Eq. (6) yields

$$\begin{aligned}&(K_A + \varepsilon\Delta K_1 + \cdots)(\Phi_A + \varepsilon\Delta\Phi) = \\ &(M_A + \varepsilon\Delta M_1 + \cdots)(\Phi_A + \varepsilon\Delta\Phi)(\Lambda_A + \varepsilon\Delta\Lambda)\end{aligned} \quad . (11)$$

Expanding Eq. (11) and comparing the terms of the same power of ε, we have

$$K_A\Phi_A = M_A\Phi_A\Lambda_A \quad , \quad (12)$$

$$K_A\Delta\Phi + \Delta K_1\Phi_A = M_A\Phi_A\Delta\Lambda + \Delta M_1\Phi_A\Lambda_A + M_A\Delta\Phi\Lambda_A. \quad (13)$$

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From Eq. (13), we have:

$$\Delta K_1\Phi_A = M_A\Phi_A\Delta\Lambda + \Delta M_1\Phi_A\Lambda_A + M_A\Delta\Phi\Lambda_A - K_A\Delta\Phi = F, \quad (14)$$

where $\Delta\Lambda, \Delta\Phi$ and ΔM are known quantities, so the stiffness matrix correction of the analytical model ΔK_1 can be obtained from this equation.

3 FRF Updating Method

An *n*-degree of freedom damped system, excited by a simple harmonic-force, can be expressed by an equation between input and output in the frequency domain, as[11]

$$X(j\omega) = H(j\omega)F(j\omega) \quad , \quad (15)$$

in which, $X(j\omega)$ is the system acceleration response, $H(j\omega)$ is the acceleration FRF, $F(j\omega)$ is the simple harmonic force. The acceleration FRF of the analysis model and the test model can be expressed as

$$H_A(j\omega) = (M_A - jC_A/\omega - K_A/\omega^2)^{-1} \quad , \quad (16)$$

$$H_t(j\omega) = (M_t - jC_t/\omega - K_t/\omega^2)^{-1} \quad , \quad (17)$$

in which M_A, C_A and K_A are, respectively, the system's mass, damping and stiffness matrices of the analysis model, M_t, C_t and K_t are those of the test model. In order to update M_A, C_A and K_A, we introduce the correction matrix as follows :

$$\begin{aligned}M_t &= M_A + \Delta M, \\ C_t &= C_A + \Delta C, \\ K_t &= K_A + \Delta K \quad ,\end{aligned} \quad (18)$$

where, ΔM, ΔC and ΔK are the corrections of the mass, damping and stiffness matrices of the analytical model. Substituting Eq. (18) into Eq. (17), we have

$$H_t(j\omega) = (Z_A + D)^{-1} \quad , \quad (19)$$

in which Z_A and D are given by the following equations:

$$Z_A = H_A^{-1}(j\omega) \quad , \quad (20)$$

$$D = -(-\Delta M + j\Delta C/\omega + \Delta K/\omega^2) \quad . \quad (21)$$

For a complex structure, only part of the structural information can be measured. Therefore, we should localize the corrections of the mass, damping and stiffness matrices on the measured degree of freedom by use of a position matrix. So we have:

$$\Delta M = PmP^{\mathrm{T}} ,$$
$$\Delta C = PcP^{\mathrm{T}} , \tag{22}$$
$$\Delta K = PkP^{\mathrm{T}} ,$$

in which m c and k are real symmetrical matrices of order $r \times r$, where r is the number of measured degrees of freedom. $P \in R^{n \times r}$ is the position matrix, which can be obtained from the $n \times n$ unit matrix by removing the columns corresponding to the non-measured degrees of freedom.

Substitution of Eq. (22) into Eq. (21) yields

$$D = P\tilde{D}P^{T} , \tag{23}$$

in which $\tilde{D}$ is defined by the following equation

$$\tilde{D} = -(-m + jc/\omega + k/\omega^{2}) . \tag{24}$$

Substitution of Eq. (23) into Eq. (19) yields

$$Z_{\mathrm{A}}H_{\mathrm{t}} + P\tilde{D}P^{\mathrm{T}}H_{\mathrm{t}} = I . \tag{25}$$

Let Eq. (25) be multiplied by $P^{\mathrm{T}}H_{\mathrm{A}}$ on the left and multiplied by P on the right, we have:

$$P^{\mathrm{T}}H_{\mathrm{A}}Z_{\mathrm{A}}H_{\mathrm{t}}P + P^{\mathrm{T}}H_{\mathrm{A}}P\tilde{D}P^{\mathrm{T}}H_{\mathrm{t}}P = P^{\mathrm{T}}H_{\mathrm{A}}P . \tag{26}$$

Substitution of Eq. (20) into Eq. (26) yields

$$P^{\mathrm{T}}H_{\mathrm{t}}P + P^{\mathrm{T}}H_{\mathrm{A}}P\tilde{D}P^{\mathrm{T}}H_{\mathrm{t}}P = P^{\mathrm{T}}H_{\mathrm{A}}P . \tag{27}$$

Assuming

$$H_{\mathrm{f}} = P^{\mathrm{T}}H_{\mathrm{A}}P , \tag{28}$$
$$H_{\mathrm{m}} = P^{\mathrm{T}}H_{\mathrm{t}}P , \tag{29}$$

substitution of Eqs. (28) and (29) into Eq. (27) yields

$$H_{\mathrm{m}} + H_{\mathrm{f}}\tilde{D}H_{\mathrm{m}} = H_{\mathrm{f}} . \tag{30}$$

Then

$$\tilde{D} = \frac{1}{H_{\mathrm{m}}} - \frac{1}{H_{\mathrm{f}}} . \tag{31}$$

Combining Eq. (31) with Eq. (24), and expanding the equation, we obtain

$$-(-m_{ij} + jc_{ij}/\omega + k_{ij}/\omega^{2}) = \tilde{D}_{ij}\ (i=1,\ \cdots,\ r;\ j=1,\ \cdots,\ i), \tag{32}$$

where m_{ij}, c_{ij} and k_{ij} are, respectively, the *ij* element of m, c and k matrices; $\tilde{D}_{ij}$ is the *ij* element of $\tilde{D}$. In the required frequency range, m_{ij}, c_{ij} and k_{ij} can be estimated by use of the curve fitting method and Eq. (32), and then by using Eqs. (22) and (18) , we obtain the updated mass, damping and stiffness matrices of the analytical model.

4 Numerical Example

Consider a damped system of 3 degrees of freedom. Its mass, damping and stiffness matrices are as follows: M_{t} = diag (28 16 16), C_{t} = diag (0.03 0.03 0.03) and K_{t} = [23 500– 8 000 0; -8 000 8 000 –5000; 0 –5 000 8 000]. The natural frequencies and acceleration FRF of this system are used as the accurate natural frequencies and acceleration FRF.

Then we give, respectively, error coefficients for K_{t}, M_{t} and C_{t} to build the original analytical model, such as, $K_{\mathrm{A}} = K_{\mathrm{t}} \times p$, $M_{\mathrm{A}} = M_{\mathrm{t}} \times q$ and $C_{\mathrm{A}} = C_{\mathrm{t}} \times r$. In this example, we assume that p=0.75, q=0.8 and r= 0.75. The natural frequencies and acceleration FRF calculated from the analytical model are considered as those to be updated. In order to validate the proposed model updating method, we input all the above model information into the program developed by using the proposed method. In the numerical example, the unit harmonic excitation with a frequency range of $\omega = 1:0.1:10(\mathrm{Hz})$ is used.

The numerical results obtained are shown in Figures 2～4, where H11, H12 and H13 are the terms of acceleration FRF matrices. In each figure, the three curves represent, respectively, the accurate FRF, the original FRF and the updated FRF.

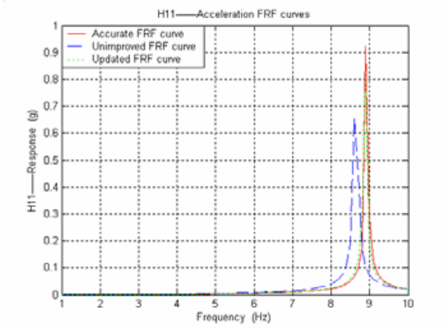

Fig.2　H11 -- Acceleration FRF curves

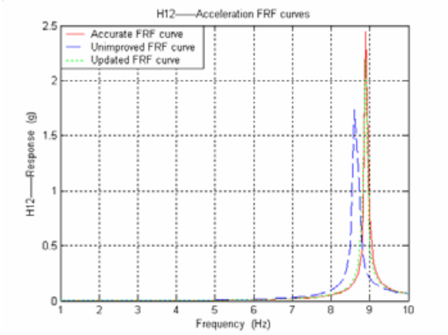

Fig.2　H12 -- Acceleration FRF curves

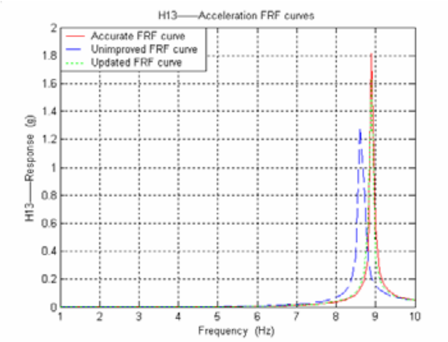

Fig.4　H13 -- Acceleration FRF curves

Table 1 shows the maximum response and the corresponding frequency for the original model, the accurate model and the updated model.

Table 1 Maximum response and corresponding frequency

	Analytical model original		Accurate model		Analytical model updated	
	Maximum Response g	Frequency /Hz	Maximum Response/g	Frequency /Hz	Maximum Response/g	Frequency /Hz
H11	0.66	8.64	0.91	8.92	0.79	8.87
H12	1.71	8.64	2.48	8.92	2.12	8.87
H13	1.26	8.64	1.81	8.92	1.60	8.87

From the above FRF curves and the numerical results in Table 1, we can see that the first natural frequency of the dynamic model is greatly improved by using the proposed method and agrees quite well with the accurate value 8.92 Hz.

In Table 1, we can also see that the maximum acceleration response of the dynamic system is considerably improved; the value of H11 is corrected from 0.66 g to 0.79 g, which is nearer to the accurate value 0.91 g. The maximum response of H12 and H13 are also improved.

From the above numerical analysis, we can conclude that the proposed method is useful for updating the frequency and the acceleration response of the analytical model.

5 Conclusions

This paper presents our studies on a synthesized model updating method by using the modal parameters and measured frequency response functions extracted from the vibration test data. The results obtained from the simple numerical model show that the proposed method can update the natural frequencies of dynamic model as well as its acceleration FRF, as a result, the acceleration FRF and the natural frequencies of the updated dynamic model are much nearer to the accurate values. However, a lot of research work remains to be done in order that the proposed method can be applied to the dynamic model of a real vibration test system. The work will mainly focus on the validity and the accuracy of this method when updating the dynamic model of the real vibration test system, and also on the investigation of the technical problem for the engineering application of the model updating program, for example, solving the interface problem between the developed program and the finite element software.

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